

# Probabilistic Teleportation of a Qudit

Arun K. Pati\* and Pankaj Agrawal†

*Institute of Physics, Sachivalaya Marg,*

*Bhubaneswar-751005, Orissa, India*

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## Abstract

It is known that if the shared resource is a maximally entangled state then it is possible to teleport an unknown state with unit fidelity and unit probability. However, if the shared resource is a non-maximally entangled state then one has to follow a probabilistic scheme where one can teleport a qubit with unit fidelity and non-unit probability. In this work, we investigate the feasibility of using partially entangled states as a resource for quantum teleportation of a qudit. We also give an expression for the probability of successful teleportation of an unknown qudit.

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\*Electronic address: akpati@iopb.res.in

†Electronic address: agrawal@iopb.res.in

## I. INTRODUCTION

Quantum teleportation protocol plays an important role in the field of quantum computation and quantum communication. This protocol allows a sender to transmit an unknown quantum state to a receiver by using an entangled state as a quantum resource and by sending classical information via ordinary channel [1]. Thus, the quantum teleportation protocol has become a paradigm example of quantum communication where the sender and the receiver are allowed to do local operations and classical communication (LOCC) only. In the original protocol, the authors showed that if the sender (Alice) and the receiver (Bob) share a maximally entangled two-qubit state, then Alice can transmit an unknown qubit with unit probability and unit fidelity to Bob. They also demonstrated how to teleport the state of a qudit (a quantum system with  $d$ -dimensional Hilbert space) with the help of a maximally entangled states of two qudits. Subsequently, the teleportation protocol has been shown to work for a wide variety of bipartite systems including when entangled states are labeled by continuous parameters [2, 3]. The protocol has also been extended and shown to work when available quantum resource is a multi-partite entangled state [4]. Experiments have also been done to demonstrate the feasibility of teleportation in laboratories [5, 6, 7].

However in real life situations, it is most of the time not possible to have a maximally entangled state at one's disposal. Because of the interaction with the environment, the state of any system would become a mixed state after a certain period. This problem of decoherence can be mitigated but cannot be completely overcome easily. Also, it may happen that the source does not produce perfect Einstein-Podolsky-Rosen (EPR) pairs rather non-maximally entangled pairs which is shared between Alice and Bob. Therefore, it is important to examine how the protocol would work with such resources. Especially, if we have non-maximally entangled state as a shared resource and we want to do quantum teleportation, then we have to pay some price. That is we have to compromise either in fidelity or in the success probability. If we are ready to pay the price for the success probability then it is possible to have unit fidelity teleportation. And this scheme we call probabilistic quantum teleportation.

It may be mentioned that in the literature, the possibility of teleportation with both types of resources have been investigated. In the case of non-maximally entangled state as a resource, there are many possibilities. Some of these possibilities are entanglement

concentration [8], use of non-maximally entangled basis [9, 10], use of POVM instead of von Neumann measurement [11, 12, 13], use of higher dimensional entangled resource [14], an antilinear operator description of the teleportation [15] and many more. One of the possibilities is that of the probabilistic teleportation when one uses non-maximally entangled basis to make von Neumann measurement. This was discovered in the case of qubits in Ref [9, 10]. This protocol has also been generalized to teleport  $N$  qubits [16]. It turns out that when we have non-ideal EPR pair like general entangled state then it is better to adopt our protocol. The probabilistic quantum teleportation scheme is a single shot on demand protocol that allows perfect teleportation of a quantum system with unit fidelity but with a probability that is less than unity. The main motive in this paper is to extend this protocol to the case of higher dimensional quantum system like an unknown qudit.

The paper is organized as follows. In Section II, we review the protocol for the case of maximally entangled quantum resource. In Section III, we present the probabilistic teleportation scheme for qubit and then generalize our results about the probabilistic teleportation to a qudit. Finally, in section IV, some conclusions are presented.

## II. TELEPORTATION WITH MAXIMALLY ENTANGLED STATE

Before discussing the teleportation with non-maximally entangled states as a quantum resource, we review the protocol for teleporting a qudit using maximally entangled states as a quantum resource. Let us consider two observers Alice and Bob possessing qudits ‘1’ and ‘2’, respectively. Their qudits are in a maximally entangled state, which can be written as

$$|\Phi\rangle_{12} = \frac{1}{\sqrt{d}} \sum_{k=0}^{d-1} |k\rangle_1 |k\rangle_2. \quad (1)$$

Alice has a qudit ‘a’ in the following unknown state

$$|\psi\rangle_a = \sum_{k=0}^{d-1} a_k |k\rangle_a, \quad (2)$$

where  $a_k$  are unknown complex numbers. Alice wishes to transmit this unknown state to Bob using local quantum operation and classical communication. To start with Alice can make a joint von Neumann measurement on her qudit ‘1’ and the qudit ‘a’. To make this measurement she can use following Bell basis for qudits

$$|\Psi_{\ell p}\rangle = \frac{1}{\sqrt{d}} \sum_{k=0}^{d-1} e^{2\pi i \ell k/d} |k \oplus p\rangle |k\rangle, \quad (3)$$

where  $k \oplus p$  means sum of  $k$  and  $p$  modulo  $d$ . The indices  $\ell$  and  $p$  can take integer values between 0 and  $d - 1$ . Here,  $|\Psi_{\ell p}\rangle$  form a set of basis vectors for a two-qudit system. We can invert this set of vectors and obtain

$$|ij\rangle = \frac{1}{\sqrt{d}} \sum_{\ell, m=0}^{d-1} e^{-2\pi i \ell k/d} \delta_{j, i \oplus m} |\Phi^{\ell m}\rangle. \quad (4)$$

Let us note that  $|\Psi_{00}\rangle \equiv |\Phi\rangle$ . Now, we can rewrite the combined state of the system ‘a12’ in terms of the above Bell basis vectors for the system ‘a1’ as follows

$$|\psi\rangle_a |\Phi\rangle_{12} = \frac{1}{d} \sum_{\ell, n=0}^{d-1} |\Psi_{\ell n}\rangle_{a1} U_{\ell n}^\dagger |\chi\rangle_2, \quad (5)$$

where these unitary operators  $U_{nm}$  are given by

$$U_{nm} = \sum_{k=0}^{d-1} e^{2\pi i n k/d} |k\rangle \langle k \oplus m|. \quad (6)$$

These unitary operators obey the following orthogonality condition

$$\text{Tr}(U_{nm}^\dagger U_{\ell p}) = d \delta_{n\ell} \delta_{mp}. \quad (7)$$

After Alice makes the von-Neumann measurement in the Bell basis (3), she would obtain one of the possible  $d^2$  results. She can convey the result of her measurement to Bob by sending  $2 \log_2 d$  classical bits of information. After receiving this information Bob uses appropriate unitary operator  $U_{nm}$  on his qudit to convert its state to that of the input state. This completes the standard teleportation protocol.

We note that here Alice succeeds in transmitting the state of the qudit ‘a’ to Bob, irrespective of the result of her measurement. So the probability of the success is unity. After receiving the information from Alice, Bob can convert the state of his qudit ‘2’ to that of the qudit ‘a’ exactly. So the fidelity of the transmitted state is also unity. In other words, it is a case of perfect teleportation. However, as we shall see, when the shared entangled resource is not maximally entangled, one has to compromise either in probability or fidelity.

### III. TELEPORTATION WITH NON-MAXIMALLY ENTANGLED STATE

We wish to now consider the situation, when available quantum resource is a non-maximally entangled two-qudit state. Earlier, a probabilistic teleportation scheme for qubit

has been proposed [9]. Here, like the case of qubit, we examine the possibility of teleportation of a qudit when the shared state is a non-maximally entangled state. We will show that it is possible to teleport a qudit with unit fidelity but with a probability that is less than unity.

### A. Probabilistic Teleportation of a Qubit

For the sake of completeness we briefly review the probabilistic quantum teleportation of a qubit. Suppose Alice receives a qubit in an unknown state  $|\psi\rangle_a = \alpha|0\rangle_a + \beta|1\rangle_a$ . She wishes to teleport this state to Bob using LOCC. However, here the pre-existing quantum channel is not a maximally entangled one, but a pure non-maximally entangled state  $|NME\rangle$  which is given by

$$|NME\rangle_{12} = N(|00\rangle_{12} + n|11\rangle_{12}), \quad (8)$$

where  $n$  is a known complex number and  $N = \frac{1}{\sqrt{1+|n|^2}}$  is a real number. Alice and Bob are in the possession of qubits 1 and 2, respectively. If Alice performs a measurement in the Bell basis on the system ‘a1’, then we know that the state  $|\psi\rangle_a$  cannot be teleported faithfully, *i.e.*, with unit fidelity and unit probability. However, if a measurement is performed in a *non-maximally entangled* (NME) basis having same amount of entanglement as that of the shared resource then it is possible for Alice to teleport the state with unit fidelity, though not with unit probability [9].

To see this, let us introduce a set of basis vectors for two qubits possessed by Alice. Using the computational basis vectors  $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$ , we can define a set of mutually orthogonal NME basis vectors as follows

$$\begin{aligned} |\varphi_\ell^+\rangle &= L(|00\rangle + \ell|11\rangle) \\ |\varphi_\ell^-\rangle &= L(\ell^*|00\rangle - |11\rangle) \\ |\psi_p^+\rangle &= P(|01\rangle + p|10\rangle) \\ |\psi_p^-\rangle &= P(p^*|01\rangle - |10\rangle) \end{aligned} \quad (9)$$

Here  $\ell$  and  $p$  are complex numbers in general and  $L = \frac{1}{\sqrt{1+|\ell|^2}}$  and  $P = \frac{1}{\sqrt{1+|p|^2}}$  are real numbers. With the change of parameter  $\ell$  and  $p$  values, this set interpolates between unentangled and maximally entangled set of basis vectors [9].

We can invert the transformations of (9) and use it to rewrite the combined state of the input and resource systems as

$$\begin{aligned}
|\psi\rangle_a |NME\rangle_{12} &= N (\alpha|0\rangle_a + \beta|1\rangle_a) (|00\rangle_{12} + n|11\rangle_{12}) \\
&= N (\alpha|00\rangle_{a1}|0\rangle_2 + \alpha n|01\rangle_{a1}|1\rangle_2 + \beta|10\rangle_{a1}|0\rangle_2 + \beta n|11\rangle_{a1}|1\rangle_2) \\
&= N [|\varphi_\ell^+\rangle_{a1}|f_1(\psi)\rangle_2 + |\varphi_\ell^-\rangle_{a1}|f_2(\psi)\rangle_2 + |\psi_p^+\rangle_{a1}|f_3(\psi)\rangle_2 + |\psi_p^-\rangle_{a1}|f_4(\psi)\rangle_2]
\end{aligned}$$

where  $|f_1(\psi)\rangle_2 = L(\alpha|0\rangle_2 + n\beta\ell^*|1\rangle_2)$ ,  $|f_2(\psi)\rangle_2 = L(\ell\alpha|0\rangle_2 - n\beta|1\rangle_2)$ ,  $|f_3(\psi)\rangle_2 = P(\beta p^*|0\rangle_2 + \alpha n|1\rangle_2)$ , and  $|f_4(\psi)\rangle_2 = P(-\beta|0\rangle_2 + \alpha np|1\rangle_2)$  are the unnormalized states. This expression is the most general way of rewriting an unknown state and two qubit entangled state.

As shown in Ref [9, 10], if Alice makes the choice  $\ell = n = p^*$ , or  $\ell = n = \frac{1}{p}$ , or  $\ell^* = \frac{1}{n} = p$ , or  $\ell^* = \frac{1}{n} = \frac{1}{p^*}$ , then for any of these choices, reliable teleportation is possible for only two out of four possible results of the measurement. For example, in the case of first choice, when the outcome is  $|\varphi_{\ell=n}^-\rangle$ , then the state at Bob's hand will be  $(\alpha|0\rangle - \beta|1\rangle)$  and when the outcome is  $|\psi_{p=n}^+\rangle$ , then the state at Bob's hand is  $(\beta|0\rangle + \alpha|1\rangle)$ . Therefore, when Alice sends two classical bits to Bob he will apply  $\sigma_z$  in the former and  $\sigma_x$  in the later case to recover the unknown state with unit fidelity. The total probability of this successful teleportation will be given by

$$P_{\text{succ}} = \frac{2|n|^2}{(1 + |n|^2)^2}. \quad (11)$$

An interesting observation in the case of qubit is that the above choice of parameters refers to the situation where the basis used for joint measurements and the resource state have the same amount of quantum entanglement, namely,  $E(|NME\rangle) = (-N^2 \log_2 N^2 - N^2 |n|^2 \log_2 N^2 |n|^2)$ . Thus, we can say that using  $E = E(|NME\rangle) = (-N^2 \log_2 N^2 - N^2 |n|^2 \log_2 N^2 |n|^2)$  amount of entanglement and two classical bits Alice *can teleport an unknown state with unit fidelity and probability given in (11)*. This is one of the main result discovered in [9].

We would like to mention two important differences between the filtering approach and ours. In the filtering approach one cannot proceed with the Bennett *et al* protocol if the filtering does not succeed. Second, in filtering approach Alice needs to communicate three classical bits (one cbit at the stage of filtering and two cbits after the Bell measurements). However, in our scheme we can carry out the protocol given any non-maximally entangled state and it works just with two cbits. That is the reason our probability of success is lower than the filtering approach.

## B. Probabilistic Teleportation of a Qudit

Let Alice and Bob share a non-maximally bipartite entangled state which is given in terms of the Schmidt decomposition form as

$$|\Phi\rangle_{12} = \sum_{j=0}^{d-1} \sqrt{\lambda_j} |j\rangle_1 |j\rangle_2, \quad (12)$$

where  $\lambda_j$ 's are the Schimdt coefficients and  $\sum_j \lambda_j = 1$ . Alice and Bob have qudits '1' and '2', respectively in the above entangled state. We wish to rewrite the above non-maximally entangled state for the sake of later convenience as follows

$$|\Phi\rangle_{12} = D \sum_{j=0}^{d-1} d_j |j\rangle_1 |j\rangle_2, \quad (13)$$

where  $D = 1/\sqrt{\sum_{j=0}^{d-1} |d_j|^2}$  is the normalization constant and  $\sqrt{\lambda_j} = D d_j$ . Alice has a qudit 'a' in the unknown state

$$|\psi\rangle_a = \sum_{k=0}^{d-1} a_k |k\rangle_a, \quad (14)$$

where  $a_k$  are unknown complex coefficients. Alice now wishes to transmit this state to Bob.

Lt us introduce a set of general two-qudit entangled states as follows

$$|\Phi^{\ell m}\rangle = N_{\ell m} \sum_{j=0}^{d-1} c_j^{\ell m} |j\rangle |j \oplus m\rangle, \quad (15)$$

where  $N_{\ell m}$  is the normalization constant and is equal to  $1/\sqrt{\sum_{j=0}^{d-1} |c_j^{\ell m}|^2}$ . If the above is part of a set of  $d^2$  orthonormal basis vectors, then the coefficients  $c_j^{\ell m}$  should satisfy the following condition, as the states  $|\Phi_{\ell m}\rangle$  would be orthonormal

$$N_{\ell m} N_{p m} \sum_{k=0}^{d-1} c_k^{*\ell m} c_k^{p m} = \delta^{\ell p}, \quad (16)$$

where the indices  $\ell, p, m$  and  $n$  take integer values between 0 and  $d - 1$ . The set of vectors  $\{|\Phi^{\ell m}\rangle\}$  will be used as a measurement basis (when appropriate condition is satisfied).

We can invert the equation (16) and obtain

$$|ij\rangle = \sum_{\ell, m=0}^{d-1} N_{\ell m} c_j^{*\ell m} \delta_{j, i \oplus m} |\Phi^{\ell m}\rangle \quad (17)$$

Let us rewrite the state of the combined of the system 'a12' as

$$|\psi\rangle_a |\Phi\rangle_{12} = D \sum_{n, j=0}^{d-1} a_n d_j |n\rangle_a |jj\rangle_{12}$$

$$\begin{aligned}
&= D \sum_{n,j=0}^{d-1} a_n d_j |nj\rangle_{a1} |j\rangle_2 \\
&= D \sum_{n,\ell,m=0}^{d-1} N_{\ell m} a_n d_{n \oplus m} c_n^{*\ell m} |\Phi^{\ell m}\rangle |n \oplus m\rangle \\
&= D \sum_{\ell,m=0}^{d-1} N_{\ell m} |\Phi^{\ell m}\rangle |f_{\ell m}(\psi)\rangle,
\end{aligned} \tag{18}$$

where  $|f_{\ell m}(\psi)\rangle = \sum_n a_n d_{n \oplus m} c_n^{*\ell m} |n \oplus m\rangle$  is a set of unnormalized kets. Note that  $|f_{\ell m}(\psi)\rangle$  has information about the unknown state. For the quantum teleportation process to succeed, on the right hand side we should have the kets  $|f_{\ell m}(\psi)\rangle$  proportional to the unknown state upto local unitary transformations, i.e.,

$$U_{nm}^\dagger |\psi\rangle = \sum_{\ell=0}^{d-1} a_\ell e^{-2\pi i n \ell / d} |\ell \oplus m\rangle. \tag{19}$$

If it is so, then after receiving classical information which is a function of  $(nm)$  Bob can apply  $U_{nm}$  to his qudit and convert its state to that of  $|\psi\rangle$ . That will complete the probabilistic quantum teleportation protocol for an unknown qudit.

From the last two equations, we notice that the condition for quantum teleportation to succeed with a finite probability is given by

$$c_n^{\ell m} = \frac{1}{d_{n \oplus m}^*} e^{2\pi i \ell n / d} = f_{n \oplus m} e^{2\pi i \ell n / d}, \tag{20}$$

where  $f_{n \oplus m}$  is a complex number with the magnitude and phase which are inverse of  $d_{n \oplus m}^*$ . As said before, if we could do this then we have

$$|\psi\rangle_a |\Phi\rangle_{12} = D \sum_{\ell,m=0}^{d-1} N_{\ell m} |\Phi^{\ell m}\rangle_{a1} U_{\ell m}^\dagger |\psi\rangle_2. \tag{21}$$

Then a von-Neumann measurement in the basis  $\{|\Phi^{\ell m}\rangle\}$  can lead to successful teleportation with unit fidelity and the probability is  $|D N_{\ell m}|^2$  for an outcome  $|\Phi^{\ell m}\rangle$ . However, as in the qubit case, not all measurements would lead to successful teleportation. This will happen only in some of the cases. To find the number of such cases, we note that the coefficients  $c_n^{\ell m}$  must satisfy the orthonormality condition (17). The successful teleportation requirement (20) may not always satisfy (17). Combining the two conditions we get

$$\sum_{n=0}^{d-1} \left( \frac{1}{|d_{n \oplus m}|^2} \right) \frac{1}{\sum_{p=0}^{d-1} \left( \frac{1}{|d_{p \oplus m}|^2} \right)} e^{-2\pi i (\ell-k)n/d} = \delta_{\ell k}. \tag{22}$$



This condition can only be satisfied when  $\ell = k$ . So the teleportation is successful only  $d$  out of  $d^2$  times. We can understand this result as follows. For a system of two qudits, these vectors  $\{|\Phi^{\ell m}\rangle\}$  naturally falls into  $d$  classes. Each class is labeled by  $\ell$ . Within each class, there are  $d$  states, which are labeled by  $m$ . These  $d$  classes are orthogonal to each other. With the choice (20) for the coefficients  $c_n^{\ell m}$ , teleportation is successful once for each class.

We can calculate explicitly the total probability of success in teleporting an unknown qudit. This is given by

$$P_{\text{succ}} = \frac{d}{\sum_{n=0}^{d-1} |d_n|^2} \frac{1}{\sum_{k=0}^{d-1} \left(\frac{1}{|d_k|^2}\right)} = \frac{d}{\sum_{k=0}^{d-1} \frac{1}{\lambda_k}} \quad (23)$$

Thus, we can say that using  $E(|\Phi\rangle) = -\sum_n \lambda_n \log_2 \lambda_n$  amount of entanglement and  $2 \log_2 d$  number of classical bits one can teleport an unknown qudit with unit fidelity but with a probability  $P_{\text{succ}}$  that is less than unity. We can check that this result reduces to the results for the qubit case. For qubit case  $d = 2$ . So one can succeed twice. This is in accord with the result of Ref [9]. As a consistent check, if we substitute appropriately for the values of  $d_n$ , then the above expression for the success probability also reduces to that of the qubit case, i.e.,  $P_{\text{succ}} = \frac{2|n|^2}{(1+|n|^2)^2}$ . Another remark is the following: In the case of probabilistic teleportation of a qubit, it was observed that the non-maximally entangled measurement basis had the same entanglement as the shared resource. However, in the case of qudit, the non-maximally entangled measurement basis do not have same amount of entanglement as the shared resource. Because, the entanglement of  $\{|\Phi^{\ell m}\rangle\}$  is  $E(|\Phi^{\ell m}\rangle) = -\sum_n N_{\ell m}^2 |c_{\ell m}|^2 \log_2 N_{\ell m}^2 |c_{\ell m}|^2$  which in general cannot be same as  $E(|\Phi\rangle) = -\sum_n \lambda_n \log_2 \lambda_n$ . Only, in the case of  $d = 2$  they coincide for the teleportation condition (20).

Furthermore, we can say that one can amplify the probability statistically by repetitions. We know that the reciprocal of the *average* success probability must be the number of repetitions  $R$  that are required in order to successfully (all the time) teleport an unknown state with unit fidelity. We see that one shall need on the average at least  $R = 1/P_{\text{succ}}$  repetitions to get a faithful teleportation with unit probability. Therefore, if Alice and Bob share  $RE(|\Phi\rangle)$  pairs of non-maximally entangled state they can successfully teleport an arbitrary qudit state using local operation and  $2R \log_2 d$  bits of classical communication. One can see that as the degree of entanglement increases, the number of required repetitions decreases and becomes one for maximally entangled states as expected. It becomes infinite for the unentangled resource state. Therefore, if Alice and Bob do not have prior shared

entanglement then it will be impossible to teleport an unknown state with unit fidelity.

#### IV. CONCLUSIONS

In this paper we have investigated how to teleport an unknown quantum state when Alice and Bob have shared a general bipartite pure entangled state in  $d \times d$ . Obviously, one cannot teleport the state with unit fidelity and unit probability. But if we pay the price for the success probability then it is possible to do quantum teleportation with unit fidelity. This we call the probabilistic quantum teleportation scheme. When the available quantum resource is not a maximally entangled state, then it is advisable to implement our scheme which is a single shot, on demand teleportation protocol without having recourse to quantum filtering or entanglement concentration. Inspired by the scheme for qubit, we have examined the possibility of teleporting the state of an unknown qudit using non-maximally entangled state as a quantum resource. We find that quantum teleportation is possible again only probabilistically, i.e., we can indeed teleport an unknown qudit with unit fidelity but with a probability less than unity. It is found that only  $d$  times out of  $d^2$  measurements, the state could be teleported with unit fidelity. We have given an expression for the success probability. We hope that with current technology one should be able to implement the probabilistic quantum teleportation protocol for a qubit and a qudit in near future. Also, one may investigate how to generalize the probabilistic teleportation protocol for continuous variable systems which seems to be a non-trivial task.

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